

Multimodal Exploration of Mathematical Function Graphs with AudioFunctions.web

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ABSTRACT

AudioFunctions.web is a web-based system that enables blind people to explore mathematical function graphs. It uses sonification, earcons and speech synthesis to convey the overall shape of a function graph, its key points of interest, and accurate quantitative information at any given point. The system can be directly linked from digital documents, such as teaching material, and it is designed to be accessed through multiple interfaces such as touchscreen, keyboard, touchpad and mouse, on both mobile devices and personal computers. This way, *AudioFunctions.web* can adapt to different user abilities, preferences and needs.

CCS Concepts

•**Human-centered computing** → *Auditory feedback; Keyboards; Pointing devices; Touch screens; Accessibility systems and tools;*
•**Social and professional topics** → *Assistive technologies; People with disabilities;*

Keywords

Visual Impairments and Blindness, Mathematics, Function graphs

1. INTRODUCTION

Bi-dimensional representations, such as function graphs, are frequently used to convey mathematical concepts. Such formalisms, however, are not accessible using traditional assistive technologies for blind people (braille displays and screen readers). Thus, studying or working in STEM fields (Science, Technology, Engineering, Mathematics) is a major challenge for blind people [2].

Traditionally, mathematical function graphs are conveyed through tactile drawings embossed on thick paper sheets. However, the amount of detail and additional information, such as labels or legends, that can be inserted within embossed drawings is limited by the paper size.

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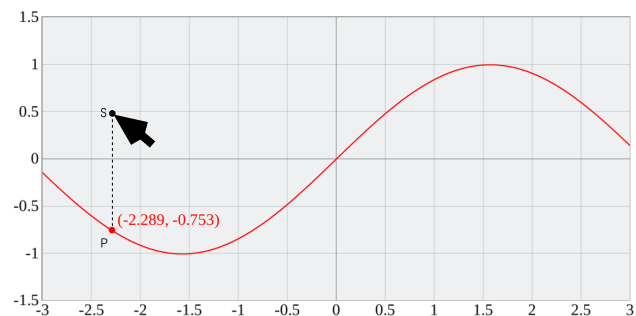


Figure 1: *AudioFunctions.web* showing a $\sin x$ function.

Ways to enrich tactile graphs with additional information have been explored [5], but the inherent limitations of this approach still apply: once a graph is embossed, the drawing cannot be modified, and therefore each update requires embossing a new sheet of paper. Previous research have also investigated alternative ways to access function graphs, using haptic force-feedback devices [3] or auditory representations [4]. However, haptic devices are cumbersome and uncommon, while prior audio-driven techniques only convey the global trend of a function graph as a fixed sound sequence.

Novel approaches combine sonification and proprioceptive touchscreen exploration [6] using off-the-shelf tablet devices. These approaches improve the construction of mental models of function graphs compared to techniques which provide sequential auditory access. Existing solutions, however, provide one-size-fits-all interfaces, and cannot adapt to different users' preferences and skills. Furthermore, they cannot be accessed directly from digital documents (e.g., a textbook PDF file), which limits their practical use.

To overcome these limitations, we present *AudioFunctions.web* [1], a web-based system that enables people with visual impairments or blindness to explore mathematical function graphs. The system combines proprioceptive touchscreen exploration and sonification feedback augmented with speech. The technique, while similar to the approach used in *AudioFunctions* [6], has been designed to be platform-independent, accessible on traditional and mobile devices through different interfaces (touchscreen, keyboard, touchpad, mouse). The interaction is further improved to provide quick understanding of global characteristics of the graph, as well as detailed exploration of its quantitative information. The software is publicly available, and can be linked directly from teaching material (e.g., PDF or web pages). Figure 1 shows an example of an *AudioFunctions.web* page.

2. AUDIOFUNCTIONS.WEB DESIGN

AudioFunctions.web enables the exploration of function graphs through sensory substitution of the function values using sonification, supported by additional verbal and sound cues. It further allows precise point exploration, conveying function values and derived quantities through verbal messages. We designed *AudioFunctions.web* as a cross platform system, accessible from different devices using touchscreen, touchpad, mouse and keyboard interfaces. We defined a set of actions required to use the system, and mapped them onto a number of corresponding interactions for each available interfaces.

2.1 Graph exploration

AudioFunctions.web draws a cartesian coordinate system which covers the whole browser window and includes the function graph within a specified value range. The scale of the graph adapts to the device resolution in order to keep the same proportions and range regardless of the device form factor. When the user moves the mouse pointer or touches the screen, the projection of the explored point onto the function graph is modulated as a sound. On keyboard, the explored point can be moved left and right using the corresponding arrow keys and the point automatically snaps onto the function graph. At any time, the user can press the escape key to return to the origin of the graph.

The produced sound varies in frequency based on the y value of the graph at the explored point. When the y value of the function is higher, also the sound played will be higher. Furthermore, the closer the explored point is to the function graph, the higher is the intensity of the produced sound. Thus, the user can track the shape of the graph by following the highest sound intensity. On stereo devices, the sound is also spatialised to reinforce the perception of the explored position. That is, it has a higher intensity on the left or right if the graph is explored on the left or right respectively. The displayed portion of the function can also be played as one sequence of sounds, from left to right, in order to have a quick grasp of the shape of the function. This is achieved through a left double click on touchpad / mouse, with a double tap on touchscreen, or with the “s” key on keyboard.

While exploring the function, the user is also notified when the explored point is a feature of interest, such as local minima / maxima, intersection with the axes or the passage through the origin of the graph. The notifications can be provided with sound cues or using verbal message. This information is designed to reinforce the understanding of the shape of the graph, and also as a cue to request additional information at those points.

2.2 Additional Information Requests

While the sonification can convey the overall structure of the graph, this information alone is not sufficient for an analytical understanding of the function and its values. Some characteristics of a function, such as its trend or its concavity are also difficult to convey through sound only. Therefore, the user can request additional information about the function at the explored point. Such information is provided through verbal messages and includes the function coordinates, its first and the second derivatives and their values at the explored point.

With touchpad or mouse, the additional information is requested through a left click at the explored point. On touchscreen, this is achieved by touching the screen with a second finger, while on keyboard, the space bar key is used. It is also possible to interrupt the reading at any time with a right click using touchpad / mouse, with the “c” key on keyboard, or simply raising the second finger on touchscreen.

3. SYSTEM IMPLEMENTATION

AudioFunctions.web is implemented using JavaScript, on top of novel web technologies and standards. It is therefore available on all modern browsers, on both mobile devices and PCs. Additionally, since *AudioFunctions.web* and its dependencies are all standalone JavaScript code, it is also possible to create a single self-contained html file which includes all the required code and can be embedded within a digital document such as PDF to be displayed offline.

3.1 Web Technologies

AudioFunctions.web draws mathematical functions graphs using *Function-Plot*¹, a javascript library for drawing function graphs, which uses SVG, HTML5 and CSS web standards to create, manipulate, style and interact with dynamic data visualizations.

To generate responsive and accurate spatialized sonification effects, *AudioFunctions.web* employs *Web Audio API*², accessed through *Tone.js* javascript library³. *Tone.js* is a framework for creating interactive music and sound effects, with advanced scheduling capabilities and musical abstractions.

For speech generation, *AudioFunctions.web* uses *Web Speech API*⁴, which provides speech synthesis and recognition capabilities.

3.2 Access from Digital Documents

AudioFunctions.web can be directly accessed from digital documents such as PDF files, ebooks or web pages. This way, it is readily available from teaching material or work documents. All the parameters used by the system can be passed as GET variables, including the function to render, the center of the coordinate system, its scale and the type of the auditory cues to use (sounds or verbal cues). The following *AudioFunctions.web* link shows the $\sin(x)$ function from Figure 1, in the range $[-5, 15]$ with musical earcons:

[https://ewserver.di.unimi.it/audiofunctions/view?f=sin\(x\)¢er=\[5,0\]&scale=20&m_earcons=1](https://ewserver.di.unimi.it/audiofunctions/view?f=sin(x)¢er=[5,0]&scale=20&m_earcons=1)

4. REFERENCES

- [1] Dragan Ahmetovic, Cristian Bernareggi, João Guerreiro, Sergio Mascetti, and Capietto Anna. 2019. *AudioFunctions.web: Multimodal Exploration of Mathematical Function Graphs*. In *International Cross-Disciplinary Conference on Web Accessibility*. ACM.
- [2] Nancy Alajarmeh. 2012. *Doing Math: Mathematics Accessibility Issues*. In *International Cross-Disciplinary Conference on Web Accessibility*. ACM.
- [3] Cristian Bernareggi, Andrea Marcante, Piero Mussio, Loredana Parasiliti Provenza, and Sara Vanzi. 2008. *Toward Haptic Mathematics: Why and How*. In *Working Conference on Advanced Visual Interfaces*. ACM.
- [4] John A. Gardner and Vladimir Bulatov. 2006. *Scientific Diagrams Made Easy with IVEOTM*. In *Int. Conf. on Computers Helping People with Special Needs*. Springer.
- [5] Timo Götzelmann. 2018. *Visually Augmented Audio-Tactile Graphics for Visually Impaired People*. *ACM Transactions on Accessible Computing* (2018).
- [6] Marzia Taibbi, Cristian Bernareggi, Andrea Gerino, Dragan Ahmetovic, and Sergio Mascetti. 2014. *Audiofunctions: Eyes-free exploration of mathematical functions on tablets*. In *Int. Conf. on Computers Helping People With Special Needs*. Springer.

¹<https://mauriciopoppe.github.io/function-plot/>

²<https://www.w3.org/TR/webaudio/>

³<https://tonejs.github.io/>

⁴<https://w3c.github.io/speech-api/>