AudioFunctions: Eyes-free Exploration of Mathematical Functions on Tablets

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Abstract. It is well known that mathematics presents a number of hindrances to visually impaired students. In case of function graphs, for example, several assistive solutions have been proposed to enhance their accessibility. Unfortunately, both hardware tools (e.g., tactile paper) and existing software applications cannot guarantee, at the same time, a clear understanding of the graph and a full autonomous study. In this paper we present *AudioFunctions*, an iPad app that adopts three sonification techniques to convey information about the function graph. Our experimental evaluation, conducted with 7 blind people, clearly highlights that, by using *AudioFunctions*, students have a better understanding of the graph than with tactile paper and existing software solutions.

1 Introduction

Visually impaired students face many difficulties while studying scientific subjects. This is due to the fact that many concepts are harder to understand without their graphical representations (e.g., a graph). In some cases, typical exercises actually require to deal with graphical information (e.g., drawing geometrical symmetries). Mathematics is probably the clearest example and indeed the hindrances of accessing this subject have been extensively studied in the literature [1]. In particular, in this paper we address the problem of rendering functions graphs accessible.

Traditional solutions to this problem include tactile drawings printed by a tactile embosser as well as on swell paper or produced with pen and sheets (e.g., by a Sewell kit). These solutions have some drawbacks. First, these drawings cannot be changed once printed, therefore the whole drawing has to be printed again to correct a mistake. Moreover, direct interaction with tactile drawings (both embossed or on swell paper) is not possible. For example a student who is given the tactile graph of a function cannot draw the symmetric function on the same drawing. Direct interaction is possible with a Sewell kit, but very good manual skills are required. One more drawback of tactile drawings concerns tactile labels (e.g. Braille labels) that are often too large to be embedded in a drawing without overlapping with lines or other labels. For example, a Braille label on the orthocentre of a triangle may easily overlap with the heights.

Software solutions have also been proposed. Among the others we mention mainstream calculus programs that can be used to generate value tables of functions (e.g. Octave, MatLab, Mathematica, etc.) and programs for graph function sonification (e.g. Audio Graphing Calculator [1] and Sonification Sandbox [2]). Value tables are very useful to get quantitative information about the functions. Nonetheless this solution presents many drawbacks. First of all, it takes a long time for a blind student to understand the curve trend. Moreover, since the exploration is basicly sequential (from left to right or from right to left along the abscissa-axis), global features such as symmetries as well as absolute maximum and minimum points can hardly be recognized by a blind student [3]. Some local features, like concavity, are very difficult to understand too.

Sonification programs enable blind students to understand the trend of a curve and the existence of local maximum and minimum points as well as intersections with the abscissa-axis [4]. Nonetheless, no quantitative information is straightforwardly provided by these programs while a blind student is exploring the curve. Hence, it turns out to be very difficult to find out global features depending on quantitative information (e.g. absolute maximum and minimum points) and features depending on exact numeric values (e.g. the coordinates of a given point such as the axes intersection points, the beginning and the end of a given interval, and so on). Moreover, the sound feedback is not enough to convey information about the asymptotic behaviour of a function (e.g. to find out an horizontal asymptote) and concavity in a given interval. One more drawback concerns the difficulty for a blind student to understand mutual relations between two distant points or between distinct portions of a curve (e.g. it is very hard to find out whether three points are on the same line or to find out symmetries).

In this paper we present *AudioFunctions*, an iPad prototype that makes it possible to overcome the problems highlighted above and that highly increases the independence of the student as well as the comprehension of the functions graphs. *AudioFunctions* presents two major improvements with respect to existing software solutions. First, being a tablet application, it benefits from the non-mediated interaction, which is typical of touchscreen devices. This in turns makes it possible to design an interaction paradigm that is based on the use of proprioception. Indeed, the second improvement consists in a set of three techniques to explore a function graph. Two of these techniques highly rely on proprioception.

We show experimentally that *AudioFunctions* tremendously improves, with respect to the other software solutions, the effectiveness of the application in terms of how clearly the user understands a function graph. Actually, *AudioFunctions* is so accurate that the testing users better recognized the function's properties with it (after five minutes of training only) rather than with tactile drawings (that every tester was well trained to use). Clearly *AudioFunctions* also has the great advantage, with respect to tactile drawings, to allow the user to study function graphs in total autonomy.

The remainder of this paper is organized as follows: in Section 2 we describe in more details the existing solutions (both hardware and software). In Section 3 we describe *AudioFunctions* and, in particular, the three techniques to explore a function graph. Our experimental evaluation is presented in Section 4 while Section 5 concludes the paper.

2 Related Work

The problem of function graphs accessibility for blind and sight impaired people has been tackled with a number of ICT solutions in the field of assistive technology for education. These solutions can be grouped in four categories: embossing tactile drawings, understanding graphs through query languages, sonification of function graphs and haptic tools for exploring diagrams.

Techniques for embossing graphs on paper have been improved over the years [1]. Indeed, at present, there are many tools to create and emboss high quality graphs on paper [5]. Nonetheless, some drawbacks still exist. First, tactile graphs cannot be edited once embossed, so the mistakes cannot be corrected and students cannot draw on the same graphs (e.g. to draw symmetric curves). In addition, Braille labels can easily overlap with lines or other labels, so they must be very short and in many situations even one letter Braille labels render difficult graph understanding [6].

The second category of techniques concerns understanding graphs through query languages. The student can understand the properties of a graph by querying information to a software application through a formal language (e.g. a calculus program such as Mathematica or Matlab) or through natural language [7]. These techniques do not provide a global understanding of the graph trend and require the student to know concepts of mathematical calculus even while exploring function graphs for the first time (e.g. what is a maximum, a minimum, concavity, etc.).

The use of sonification to enable sight impaired people to access function graphs has been studied through Audio Graphing Calculator [1] and Sonification Sandbox [2] applications. These sonification programs provide a sound description of a function graph by using simple sound [1] and midi sound [2]. Sonification enables blind students to understand the trend of the graph and relevant points such as maxima, minima and intersections [4]. Unfortunately, no quantitative information is straightforwardly provided by these programs while a blind student is exploring the curve. Moreover, the sound feedback is not enough to convey information about the asymptotic behaviour of a function (e.g. to find out an horizontal asymptote) and concavity in a given interval. One more drawback concerns the difficulty for a blind student to understand mutual relations between two distant points or between distinct portions of a curve (e.g. it is very hard to find out whether three points are on the same line or to find out symmetries).

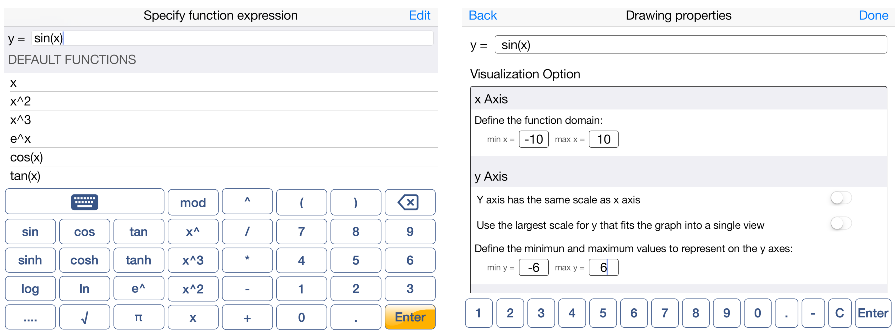
Finally, haptic and audio-haptic systems have also been proposed to make possible non-visual graph exploration and manipulation [8]. The main advantage of these systems consists in the ability of touching and manipulating the graph. Moreover, guided exploration is also possible. The hand of the student is guided by the arm of an haptic device (e.g. the Phantom) along the curve. Unfortunately, haptic devices are still very expensive. Moreover, the workspace is limited (about 15 by 15 cm), so graphs with many details can be hardly understandable.

3 The *AudioFunctions* Prototype

The use of *AudioFunctions* can be divided in two main activities: the specification of the function expression as well as of its drawing properties and the function graph exploration.

3.1 Specification of Function Expression and Drawing Properties

To specify a function expression, a user can choose a template and then edit it (see Figure 1(a)). The template can be chosen from the list of "default" expressions (i.e., a pre-defined set of common functions, like $y = x$, $y = x^2$ or $y = sin(x)$) or from the list of recently used expressions, as they were edited by the user (in Figure 1(a) the list of recently used expressions is hidden by the keyboard). To edit the function expression, *AudioFunctions* presents an ad-hoc keyboard, that is similar to a calculator keyboard and that contains keys for the digits and for the most common arithmetic and trigonometric operators.



(a) Specification of function expression (b) Specification of drawing properties

Fig. 1. Specification of the function expression and its drawing properties

The function drawing properties (see Figure 1(b)) include options to define the domain and the scale on the two axes. With the first option, the user can set the function domain in terms of the minimum and maximum values of x to be represented. The second property is a boolean value indicating if the y axis should have the same scale as the x axis. If this property is set to "true" (the default value) then the next two options are disabled. In case this property is set to "false", with the third option the user can choose to automatically scale the y axis, which means that *AudioFunctions* chooses the largest scale for the y axis such that the function graph fits in the screen. If "automatic scale" is disabled, then the user can manually choose the scale for the y axis, by indicating the minimum and maximum values to represent on the y axis.

3.2 Function Graph Exploration

The exploration of the function graph supports three "exploration modes" (see Figure 2). The first one, that we call "non interactive", is analogous to the solution proposed in Audio Graph Calculator and Sonification Sandbox: by using a "double two finger tap" gesture[3] *AudioFunctions* starts playing the function sonification, which is obtained as follows. As shown in Figure 2(a), *AudioFunctions* divides the function domain into a set of intuitively small intervals (e.g. the rectangle r). For each interval, given the sonification direction starting from the lowest and up to the highest x coordinate, the app computes the value of $y = f(x)$ where x is the minimum value of the interval and reproduces the "value-sonification" for y, i.e., a sound whose pitch is proportional to the value of y with respect to the range of y values.

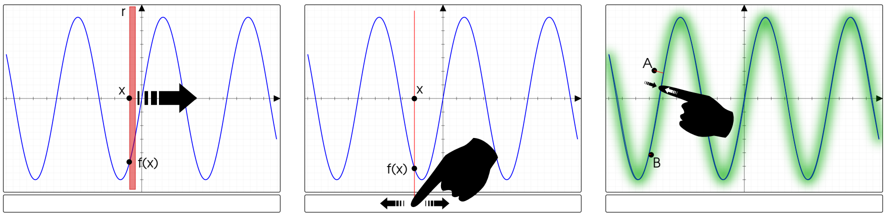
Example 1. With the function $y = sin(x)$, for $x \in [-10, 10]$, when $x = \pi/2$ we have $y = 1$ which is also the maximum value for y and hence the value-sonification for $x = \pi/2$ has the highest pitch. Vice versa, if we draw $y = x$, for $x \in [1, 10]$, the sonification for $x = 1$ has the lowest pitch, because 1 is the smallest value represented for y.

We call the second exploration mode "mono-dimensional interactive" (Figure 2(b)). The user can slide the finger along an horizontal bar positioned at the bottom of the view that represents the x axis.

While sliding the finger, *AudioFunctions* uses the value-sonification technique to represent the value $y = f(x)$ where x corresponds to the current finger position. The clear advantage of this exploration mode is that, thanks to proprioception, the user can perceive the current x position. Also, the user can move forward and backward along the x axis, at the desired speed, hence, for example, focusing more on some parts of the functions that are more relevant for the user (e.g., a minimum point).

The third exploration mode is called "bi-dimensional interactive". The overall idea is to make it possible for the user to follow with one finger the graphical representation of the graph. The shape of the graph is then perceived thanks to proprioception. This mode adopts a different sonification, that we call "position-sonification", since the aim is not to encode the y value, rather to guide the user while following the plotted line. When the user touches on the function line, the position-sonification reproduces a sound with the highest pitch. When the user touches outside the line, the pitch diminishes as the minimum distance between

[3] This is the gesture that on iOS devices is associated, for example, to start and pause music reproduction.



(a) Non interactive (b) Mono-dimensional interactive (c) Bi-dimensional interactive

Fig. 2. Function exploration screen and Exploration modes

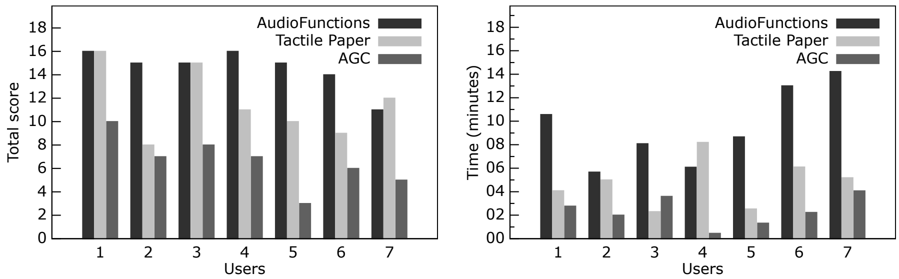
the touched position and the line increases. For example, in Figure 2(c), point A is more distant from $f(x)$ than point B. Therefore, when the user touches A a low pitch sound will be played while touching B will yield a high pitch sound.

The two interactive modes have some additional features. First, while exploring, *AudioFunctions* reproduces some additional sounds in case the function intersects some "interesting points", like intersection with the axis, local minimum and maximum and changes in the concavity. Second, interaction with two fingers is supported. This is very useful, for example, when it is necessary to maintain a reference point in the exploration. To achieve this, when a second finger touches the screen, *AudioFunctions* starts reproducing the sound associated to that finger, ignoring the first one. Third, by double tapping, *AudioFunctions* reads details on the current position, including: the values of x and $f(x)$ and the function concavity in that point. This is useful because function concavity is easily understandable by sight, but hard to figure out with these sonification techniques.

4 Experimental Evaluation

The main objective of *AudioFunctions* is to let the user perceive the shape of a function graph. Therefore we focused our experiments to determine how precisely a user can recognize the function properties from the exploration. To measure the level of understanding of the function, we asked each user to explore the graph and to answer 8 questions (e.g., "which is the concavity of the function for $x = 0$?"). We scored each answer with a mark of 0 (totally wrong answer or no answer), 1 (partially correct answer) or 2 (correct answer). We run the experiment with 7 blind users, all with some education in Mathematics (at least high school) and acquainted with tactile drawings. During each test session we first described *AudioFunctions* in about 2 minutes and then we left 3 minutes to let the user get familiar with the app. After these 5 minutes training we started the test that was divided into three steps, each one involving a different tool: *AudioFunctions*, tactile drawings and "Audio Graphing Calculator" (AGC). The order of the three steps was random. During each step we chose a random function expression from a set of pre-defined functions presenting the corresponding graphs to the user and posing him/her the 8 questions. While answering each question the user was free to interact with the exploration tool. We recorded the answers and the time needed to provide them.

Figure 3(a) shows, for each user and technique, the sum of the scores obtained in all the questions (we recall that the maximum is 16). Intuitively this metric represents the overall understanding of the function obtained by each user with each technique. We can observe that every user obtained much better results by using *AudioFunctions* with respect to AGC. *AudioFunctions* also proved to be more effective also compared with the tactile drawings that, we recall, all the users were acquainted with. Indeed, every user, except user 7, obtained better results with *AudioFunctions* than with tactile drawing and for most of the users the results with *AudioFunctions* are much better than with tactile drawings (e.g., users 2, 4, 5 and 6).



(a) Function comprehension (b) Total time to answer

Fig. 3. Results of the experimental testing.

Figure 3(b) compares the total time required by each user to answer the 8 questions by using each technique. Results show that, by using AGC, users provided answers more quickly (about 2 minutes on average) than with tactile drawings (about 5 minutes on average) and *AudioFunctions* (about 9 minutes on average).

5 Conclusions and Future Work

In this paper we presented *AudioFunctions* a tablet prototype that allows visually impaired students to explore function graphs. *AudioFunctions* presents three exploration modes, two of which use propriception by taking benefit from the direct interaction with the tablet touchscreen. The experimental evaluation that we conducted with 7 users shows that *AudioFunctions* allows the users to have a much better understanding of the function graph than existing software solutions. *AudioFunctions* allows the users to have a better understanding also when compared to tactile paper. This was not expected, as the users only trained with *AudioFunctions* for a few minutes, while they were all acquainted with mathematical exercises on tactile paper.

While the aim of *AudioFunctions* was to explore different interaction paradigms, as a future work we intend to focus on the sonification technique, to compare different solutions and identify the one that best suites each exploration mode. We also plan to engineer *AudioFunctions* and distribute it on the AppleStore. This would allow a large distribution of the app, which in turn can have positive effects on future research. Indeed, by remotely collecting usage data, it could be possible to evaluate the solution with a much larger number of users, possibly in the order of hundreds or thousands.

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